

Data Science and Statistical Computing

Tutorial 5, Week 10

Q1

In an example in lectures we showed how to inverse sample from a standard Cauchy distribution, and used these samples to rejection sample from a standard Normal distribution. We will try another approach to sample standard Normals here.

- a) The standard Laplace distribution (sometimes called the double exponential distribution) has pdf:

$$\tilde{f}(x) = \frac{1}{2} \exp(-|x|) \quad \forall x \in \mathbb{R}$$

Derive an inverse sampling scheme to generate simulations from this distribution.

- b) If $f(x)$ is the pdf of a standard Normal distribution, show $\exists c < \infty$ such that

$$f(x) \leq c\tilde{f}(x) \quad \forall x \in \mathbb{R}$$

and find the smallest such c .

- c) Write down the steps to produce a rejection sampled simulation from a standard Normal distribution via an inverse sampled standard Laplace simulation.
- d) In lectures, the standard Cauchy distribution had $c \approx 1.521$ when used to rejection sample standard Normals. In light of your analysis, would you favour simulation via the standard Cauchy or standard Laplace and why? How much better is your choice than the alternative.

Q2

Let X be a random variable on \mathbb{R} with cdf $F(\cdot)$. The so-called *truncated* version of a random variable arises when we restrict the support of X to some range $X \in [a, b] \subset \mathbb{R}$. In other words, it is the random variable $(X \mid a \leq X \leq b)$

- a) Prove that the cdf of the truncated random variable is:

$$\mathbb{P}(X \leq x \mid a \leq X \leq b) = \begin{cases} 0 & \text{if } x < a \\ \frac{F(x) - F(a)}{F(b) - F(a)} & \text{if } a \leq x \leq b \\ 1 & \text{if } x > b \end{cases}$$

- b) Hence or otherwise, find an inverse sampler for any truncated distribution in terms of $F(\cdot)$.
- c) Write down an inverse sampler for an Exponential distribution truncated to $[1, \infty)$
- d) Does the result in (c) confirm any properties of the Exponential distribution?