Data Science and Statistical Computing

Assignment 4

Due Monday 9th December 2024 at noon in Gradescope

The Exponential distribution has probability density function (pdf):

$$\tilde{f}(y \mid \lambda) = \begin{cases} \lambda e^{-\lambda y} & \text{if } y \in [0, \infty) \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$.

1. Simulate three values (pen-and-paper, not R) from this pdf via inverse transform sampling using the following values simulated from the Uniform$(0, 1)$ distribution:

$$0.56, \;\; 0.85, \;\; 0.26$$

Let the random variable X have pdf:

$$f(x \mid \mu) = \begin{cases} \mu^2 x e^{-\mu x} & \text{if } x \in [0, \infty) \\ 0 & \text{otherwise} \end{cases}$$

where $\mu > 0$.

2. Show that the Exponential distribution can be used as a proposal distribution in a rejection sampler to generate simulations of X. Ensure you state any conditions on λ and μ.
3. For any choice of μ, what is the optimal λ to choose as the parameter in the proposal distribution?
4. Show that when the optimal λ is used the expected number of iterations required to produce a single simulation of X is approximately 1.47 for all μ.